ABSTRACT

        Commonality analysis may be used as an adjunct to
general linear methods as a means of determining the degree of
predictive ability shared by two or more independent variables. For
each independent variable, commonality analysis indicates how much of
the variance of the dependent variable is unique to the predictor and
how much of the predictor's explanatory power is common to or also
available from one or more of the other predictor variables.
Commonality analysis is particularly useful in social science
research involving multivariate data sets with at least one predictor
at the interval level of scale, since, unlike many analyses of
variance techniques, it does not require that all the independent
variables be converted to the nominal level of scale. A small,
hypothetical data set is presented to illustrate the value of
commonality analysis and to demonstrate its usefulness in
interpreting results from educational experiments using both
univariate and multivariate methods. The statistical examples
provided serve as models to researchers of ways of implementing
commonality analysis as an adjunct to various univariate and
multivariate statistical methods. Ten data tables are provided.
(TJH)

COMMONALITY ANALYSIS WITH MULTIVARIATE DATA SETS

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ABSTRACT

Commonality analysis may be used as an adjunct to general linear methods as a means of determining the degree of predictive ability shared by two or more independent variables. The strengths of commonality analysis are discussed. A small, hypothetical data set is presented to illustrate the value of commonality analysis, and to demonstrate its usefulness in interpreting results from educational experiments using both univariate and multivariate methods. The statistical examples provided will serve as models to researchers of ways to implement commonality analysis as an adjunct to various univariate and multivariate statistical methods.

COMMONALITY ANALYSIS WITH VARIOUS PARAMETRIC METHODS

Experimental data can often be analyzed in a number of ways, with various analyses yielding different results. According to several reviews of statistical methods used in published educational research (Elmore & Woehlke, 1988; Goodwin & Goodwin, 1985a, 1985b; Willson, 1980), analysis of variance (ANOVA) methods and their various analogs (i.e., ANCOVA, MANOVA, MANCOVA)--collectively labelled here as "OVA methods" (Thompson, 1985)--are used more frequently than any other statistical technique. Willson (1980), for instance, found that OVA methods accounted for nearly 41 percent of the statistical techniques used in articles published in the <u>American Educational Research Journal</u> from 1969 to 1978. From 1979 to 1983, OVA techniques were used in approximately a third of the articles published in <u>AERJ</u> (Goodwin & Goodwin, 1985b). Similarly, Elmore and Woehlke (1988) reported that ANOVA and ANCOVA accounted for 25 percent of statistical methods used in research published in three educational research journals during the years 1978 through 1987. Thus, there is some trend away from the used of OVA methods in educational research, although these methods are still used with considerable frequency.

One of the problems with the widespread use of OVA methods is that it is necessary to reduce intervally scaled predictor variables to nominal categories whenever predictor variables are originally intervally scaled. A researcher might, for example,

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convert interval scores on a given measure into three levels (low, medium, high) in order to create proportional OVA cells, each containing an equal number of cases. In reducing the level of scale of predictor variables, researchers not only throw away data that they have gone to a lot of trouble to collect, but may also distort the reality underlying the original data (Kerlinger, 1986; Thompson, 1986a). Furthermore, reduction of the original level of scale of predictor variables may adversely affect the variables' reliabilities (Cohen, 1968).

Other general linear methods (i.e., multiple regression analyses, canonical correlation analyses) are often superior to OVA techniques in that they maintain the interval level of scale of predictor variables (Thompson, 1985). Considering this fact, it is not surprising that, in some research situations, the more general linear methods have been shown to be superior to OVA methods in accurately estimating explained dependent variable variance (e.g., Thompson, 1986a).

Despite this advantage of the more general methods, OVA methods do allow the researcher to divide dependent variable variance into a number of partitions, including the interaction effects of any two or more predictor variables interacting together as they affect the dependent variable. For example, in the three predictor variable case, the researcher using a full factorial ANOVA is able to investigate the main effects of each predictor variable (A, B, and C), the unique two-way interaction effects of each pair of predictor variables (A by B, B by C, and

A by C), and the three-way interaction effect of all three predictor variables (A by B by C) as they affect the dependent variable D. However, although many researchers do not realize it, these effects can also be investigated with non-OVA general linear methods.

Use of Product Variables to Represent Interaction Effects

Kerlinger and Pedhazur (1973) discuss a method whereby "product variables" can be used to represent unique interaction effects among a given set of predictor variables. In their simplest form, product variables are the multiplicative product of any two or more variables. Researchers can make use of these product variables to represent interaction effects using more general analytic models. For example, in the three predictor case, using predictors A, B, and C in their original levels of scale, four product variables V(x) could be computed:

$$V(1) = A * B$$
$$V(2) = A * C$$
$$V(3) = B * C$$
$$V(4) = A * B * C$$

These product variables could then be used in a regression or canonical equation to represent the interaction effects of the predictors acting together to explain the variance of the dependent variable(s). Hence, by using product variables, researchers may test for interaction effects among variables, and simultaneously maintain the level of scale of predictor variables.

Using Commonality Analysis to Determine Variable Contribution

Commonality analysis has been defined as "an attempt to understand the relative predictive power of the regressor variables, both individually and in combination" (Beaton, 1973, p. 2). Using commonality analysis, a researcher can determine the unique and the common contributions of each independent variable and each interaction effect in a prediction equation. As Thompson (1985) states:

> For each independent variable, commonality analysis indicates how much of the variance of the dependent variable is "unique" to the predictor, and how much of the predictor's explanatory power is "common" to or also available from one or more of the other predictor variables. (p. 53)

Originally suggested by Kempthorne (1957), commonality procedures have been explored by a host of researchers (e.g., Beaton, 1973; Creager & Valentine, 1962; Mayeske, Wisler, Beaton, Weinfield, Cohen, Okada, Proshek, & Tabler, 1969; Newton & Spurrell, 1967a, 1967b; Seibold & McPhee, 1979; Thompson, 1985, 1988; Thompson & Miller, 1985). Newton and Spurrell (1967a, 1967b) referred to the procedure as "element analysis," while Mayeske et al. (1969) used the label "components analysis." The more familiar term "commonality analysis" was first used by Mood (1971).

A commonality analysis for a one dependent variable case is conducted by first running a series of regression analyses using

every possible unique combination of independent variables as predictors. The multiple R obtained in each of these analyses is, in turn, subtracted from the multiple R for the analysis using all of the predictors to determine the degree of unique variance explained by each variance partition.

Thompson (1985) demonstrated the advantages of commonality analysis over two commonly used OVA methods using a small hypothetical data set involving two predictors and one dependent variable. The first two analytic methods employed were a two-way factorial ANOVA and a two-way ANOVA via regression coding. The regression coding method was shown to be superior to the original ANOVA as it succeeded in breaking down effects into smaller comparative units, yielding more specific information about where differences occured within the analysis. Commonality analysis was shown to be more powerful still, as it not only served to show the effect of each partition of the explained variance, but also maintained the level of scale of the original data.

Commonality Analysis in the Multivariate Case

Darlington, Weinberg, and Walberg (1973) discussed the appropriateness of conducting canonical variate analyses in research situations involving relations between two sets of variables, where the size of each data set is ≥ 2. Similarly, Thompson (1986b) demonstrated how studying the interaction of sets of related variables can have an impact on the interpretation of statistical results. Yet, the unique explanatory power of individual independent variables in a

multivariate data set is also a significant issue. Commonality analysis can also be useful in the multivariate case to address this issue.

To date, most of studies investigating the usefulness of commonality analysis (e.g., Mood, 1971; Newton & Spurrell, 1967a, 1967b; Thompson, 1985) have focused upon the one dependent variable case. However, as previously noted, commonality analysis may also be appropriately used with multivariate (i.e., two or more dependent variable) data sets when at least one predictor is at the interval level of scale (Thompson, 1988; Thompson & Miller, 1985). The multivariate application of commonality analysis is particularly valuable in social science research as many variables worthy of experimental study are highly correlated with one another. Commonality analysis can be an invaluable aid to the researcher who must analyze multivariate data sets containing theoretically or empirically distinct sets of variables (Thompson, 1988). Beaton (1973, p. 38) recognized the importance of multivariate commonality analysis, stating:

> Multivariate commonality is a technique for assessing the common and unique predictability of several regressors or sets of regressors on a set of $p \geq 1$ regressands. The technique is a simple generalization of univariate commonality and the results of the two will be the same if the value of p is unity. Multivariate commonality is to multivariate analysis very much as univariate

commonality is to regression analysis.

Multivariate commonality analysis differs from its univariace analog in that the several criterion variables must be transformed into a composite variable prior to conducting the necessary regression analyses. Prior to these transformations, all of the criterion variables must be converted to a standard metric, e.g., z-score form (Beaton, 1973). Once these conversions are made, standardized canonical function coefficients may be used to weight the dependent variables (Thompson & Miller, 1985). These weighted variables are then added together to create dependent variable composites to be used in standard regression analyses.

A Hypothetical Data Set

The fictitious data set employed for the present study consists of three predictor and two criterion variables. These data are presented in the first seven columns of Table 1. The hypothetical research situation was an educational experiment designed to determine which of two classroom settings was most appropriate for a sample (N = 16) of "low" and "high" IQ students. Eight boys and eight girls were included in the study. Original IQ data were converted into OVA categories to allow for the use of ANOVA and MANOVA analyses. Each of the three predictor variables (SEX, OVAIQ, and GROUP) was comprised of two levels, yielding a total of eight balanced cells (2 X 2 X 2), each containing two subjects. The predictor variables were comprised of the following levels:

VARIABLES	LEVELS
OVAIQ	1 = "LOW" 2 = "HIGH"
GROUP	1 = "CONTROL" 2 = "EXPERIMENTAL"
SEX	1 = "MALE" 2 = "FEMALE"

Dependent variables in the study were a reading posttest (DV1) and a measure of the students' attitude toward reading (DV2). These variables were scaled as follows:

DV1--Posttest Score   (LOW) 1 2 3 4 5 6 . . .100 (HIGH)

DV2--Attitude Measure   (LOW) 1 2 3 4 5 6 7 8 9 (HIGH)

INSERT TABLE 1 ABOUT HERE

Analysis of the Data

Data were analyzed using four different statistical methods: (1) ANOVA, (2) univariate commonality analysis, (3) MANOVA, and (4) multivariate commonality analysis. The first two analyses (ANOVA and univariate commonality analysis) utilized all three predictor variables and one dependent variable (DV1). The SPSSx command file for running these analyses is presented in Appendix A. Analyses (3) and (4) utilized the three predictors and both dependent variables (DV1 and DV2). The SPSSx command file for running these analyses is presented in Appendix B. The four analyses can be compared to evaluate the utility of different statistical choices. Results of each analysis are discussed below.

ANOVA Analysis

Data were analyzed first using a three-way factorial ANOVA.

Two of the ways (sex and experimental condition) were already at the nominal level of scale. Since the remaining way (IQ) was intervally scaled, it was converted to the nominal level as well to perform the ANOVA. The reading posttest score (DV1) served as the dependent variable. The results of this analysis are presented in Table 2. Effect sizes for each predictor were computed by dividing the sum of squares for each effect by the sum of squares total. Due in part to the small sample employed in this hypothetical study (Carver, 1978), none of the effects was statistically significant using an alpha level of .05. The source variables accounted for 42.2 percent of the variance, with IQ and the three-way interaction effect accounting for the majority of the explained variance.

INSERT TABLE 2 ABOUT HERE

Univariate Commonality Analysis

As previously noted, regression and commonality analysis is a helpful statistical combination as it allows for the testing of interaction effects while maintaining the original level of scale at which data were collected. To illustrate this point, a commonality analysis was conducted using the same data analyzed in the previous ANOVA procedure. Both main and interaction effects were assessed in the commonality regression procedure. Variables were converted to z-scores prior to performing this analysis, and product variables were computed to represent the interaction effects among the predictor variables. These Z-score

conversions and product variables are presented in Table 1.

Results of regression analyses using various different predictor sets are presented in Table 3. The predictor sets used in these analyses were determined somewhat arbitrarily to consist of (1) all main effects, (2) all two-way interactions, (3) the three-way interaction, (4) and all combinations of (1), (2), and (3). However, in an actual study using real data, the researcher would probably wish to predetermine variable sets based upon theoretical or intuitive relationships among the variables. Ideally, _every_ possible variable combination would be tested; however, for ease of explanation only seven possible sets are considered here.

INSERT TABLE 3 ABOUT HERE

Interestingly, the multiple correlation coefficient (R^2) using all three of the predictor variables and all possible interaction effects (predictor set 7) to predict the dependent variable was .567 (effect size = 56.7 percent). The total effect using the previous ANOVA analysis was only 42.2 percent. The difference in these values illustrates well what may happen when interval predictor variables are converted to the nominal level of scale in order to conduct an OVA analysis. The researcher using the ANOVA in this case would have underestimated the actual effect by more than 14 percent!

Table 4 presents the calculations necessary to determine the unique contribution of each of the selected variance partitions

to the predictive equation. Commonality analysis results are presented in Table 5. These data indicate that the main effect independent variables account for the greatest portion of explained variance when used in isolation. The three two-way interactions account for about two percent less of explained variance when used in isolation. It is noteworthy that the single three-way interaction accounts for nearly 20 percent of the variance if used in isolation. In ordinary regression analysis, this predictor would not have been utilized. It is also interesting to note that all but one of the commonality values in Table 4 are negative. Although these values may appear to indicate that the given sets of predictor variables have in common the ability to explain less than 0 percent of the variance, these values actually indicate the presence of suppressor effects in the variable sets (Craeger, 1971; Thompson, 1985), since variances can of course never be negative.

<u>INSERT TABLES 4 AND 5 ABOUT HERE</u>

MANOVA Analysis

In the third statistical analysis run on the present study's hypothetical data set, a second dependent variable (DV2) was added. Data were analyzed using a multivariate analysis of variance. The results of this MANOVA are presented in Table 6. Multivariate effect sizes were small to moderate for each of the seven partitions tested with the main effects of experimental condition (GROUP) and IQ (OVAIQ) each explaining over 30 percent of the variance across the two dependent variables (as indicated

by the value of Wilks' lambda at each successive step). None of
the multivariate or univariate tests yielded statistically
significant results at the .05 level.

INSERT TABLE 6 ABOUT HERE

Multivariate Commonality Analysis

The final statistical analysis of the hypothetical data set
was performed via canonical correlation and multivariate
commonality analyses. The canonical correlation analysis yielded
a squared canonical correlation coefficient (R_c^2) of .42573
(Wilks' lambda = .53405; F = 1.35073 (df = 6,22)). Standardized
canonical function coefficients were:

Variable	Function I	Function II
DV1	.94102	.49277
DV2	.78126	-.71970

Composite scores for the dependent variables were computed
using these canonical function coefficients. These composite
values are listed in Table 7. Next, regression analyses were
utilized to determine the commonality values of each explainable
variance partition of the dependent variable composite resulting
from the Function I weights. The results of the regression
analyses using alternate predictor variable combinations to
predict the canonical composite dependent variable are presented
in Table 8. A full commonality analysis was completed here for
the Function I composite variables only, although the results
from the Function II composite regression equations could be

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easily interpreted as well.

<u>INSERT TABLES 7 AND 8 ABOUT HERE</u>

Squared bivariate correlation coefficients indicated that both ZGROUP and ZIQ are effective in explaining at least 20 percent of the dependent variable variance when used singly as predictors. Although ZSEX is not a very good predictor when used singly, it is interesting to note that when used in combination with ZIQ (predictor set 5), the total explanatory power of the two variables together is almost 33 percent. The total explanatory power of all the variables used together (predictor set 7) is 42.573 percent, as determined from the originally derived squared canonical correlation coefficient ($R_c^2 = .42573$). The regression result from predicting the canonical composite score for the dependent variable set with all predictors should always equal the canonical result, since the two analyses are in fact the same.

Table 9 presents the calculations necessary to determine the unique contribution of each of the variance partitions to the predictive equation. Commonality analysis results are presented in Table 10. Results indicate that ZGROUP and ZIQ share the most common predictive power, and that of the three predictors, ZIQ has the most unique predictive power. Considering the predictive power of this variable, it is most important that a statistical method was used which did not reduce the variable's original level of scale. Suppressor effects are noted for two of the commonality values in Table 9 (ZIQ/ZSEX and ZGROUP/ZSEX/ZIQ).

Interestingly, both of these variable combinations include the variable ZSEX, which has the least predictive power of any of the three predictor variables.

<center>INSERT TABLES 9 AND 10 ABOUT HERE</center>

Discussion

Commonality analysis offers the educational researcher an effective method for interpretation of individual variance partitions within general linear models. However, "[t]he compelling advantage of commonality analysis of experimental data is that the analysis does not require that all independent variables be converted to the nominal level of scale" (Thompson, 1985, p. 54). This important aspect of commonality analysis gives it power over analyis of variance techniques which often necessitate the reduction of interval data into nominal categories. Hence commonality analysis provides the researcher with a valuable technique for breaking down dependent variable variance into a number of partitions without distorting the reality of the data.

Commonality analysis is particularly of value in multivariate cases since multiple dependent variables are frequently of interest and since social science variables worthy of experimental study are frequently highly correlated with one another. When data sets contain predictor variables that are higher than nominally scaled, or when data sets include theoretically distinct sets of variables, commonality analysis is

particularly useful. In the present analyses, general linear techniques using commonality methods were compared with traditional OVA methods. Researchers would do well to investigate other applications of commonality analysis as has been suggested by Beaton (1973).

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Table 1

Hypothetical Data Set

CASE	GROUP	SEX	IQ	OVAIQ	DV1	DV2	Z-score conversions					Commonality Data			
							ZDV1	ZDV2	ZGROUP	ZSEX	ZIQ	ZIQBYZGP	ZSXBYZGP	ZIQBYZSX	THREEWAY
1	1	1	93	1	18	8	-1.92	1.30	-.97	-.97	-.63	.61	94	.61	-.59
2	1	2	88	1	84	2	.49	-1.05	-.97	.97	-.99	.96	-.94	-.96	.93
3	2	1	85	1	64	5	-.24	.12	.97	-.97	-1.21	-1.17	-.94	1.17	1.13
4	2	2	95	1	81	3	.38	-.66	.97	.97	-.49	-.47	.94	-.47	-.46
5	1	1	93	1	98	5	1.00	.12	-.97	-.97	-.63	.61	.94	.61	-.59
6	1	2	95	1	55	9	-.57	1.69	-.97	.97	-.49	.47	-.94	-.47	.46
7	2	1	85	1	49	1	-.79	-1.45	.97	-.97	-1.21	-1.17	-.94	1.17	1.13
8	2	2	87	1	14	5	-2.07	.12	.97	.97	-1.07	-1.03	.94	-1.03	-1.00
9	1	1	130	2	99	3	1.04	-.66	-.97	-.97	2.03	-1.96	.94	-1.96	1.90
10	1	2	117	2	84	8	.49	1.30	-.97	.97	1.09	-1.06	-.94	1.06	-1.02
11	2	1	118	2	47	7	-.86	.91	.97	-.37	1.16	1.13	-.94	-1.13	-1.09
12	2	2	106	2	99	1	1.04	-1.45	.97	.97	.30	.29	.94	.29	.28
13	1	1	118	2	83	6	.46	.51	-.97	-.97	1.16	-1.13	.94	-1.13	1.09
14	1	2	112	2	81	4	.38	-.27	-.97	.97	.73	-.71	-.94	.71	-.69
15	2	1	103	2	74	2	.13	-1.05	.97	-.97	.09	.08	-.94	-.08	-.08
16	2	2	104	2	99	6	1.04	.51	.97	.97	.16	.15	.94	.15	.15

Table 2

ANOVA Analysis

Source	Sum of Squares	df	Mean Square	F-calc	Effect Size
OVAIQ	2575.6	1	2575.6	3.18	23.0%
GROUP	351.6	1	351.6	.43	3.1%
SEX	264.1	1	264.1	.33	2.4%
OVAIQ by GROUP	22.6	1	22.6	.03	.2%
OVAIQ by SEX	189.1	1	189.1	.23	1.7%
GROUP by SEX	175.6	1	175.6	.21	1.6%
Three-way	1139.1	1	1139.1	1.41	10.2%
Error	6474.5	8	809.3		
TOTAL	11192.0	15	746.1		42.2%

Table 3

Prediction of Dependent Variable Using

Alternate Predictor Variable Combinations

Predictor set	Variable(s) in set	R^2
1	ZIQ, ZGROUP, ZSEX	.233
2	ZIQBYZGP, ZSXBYZGP, ZIQBYZSX	.020
3	THREEWAY	.106
4	1 AND 2	.284
5	1 AND 3	.380
6	2 AND 3	.137
7	1, 2, AND 3	.567

Table 4

Calculations of Unique Variance Partitions

Partition	Result
Unique to ZIQ, ZSEX, ZGROU (1) −R sq 6 + R sq 7 −.13730 + .56680	.42950
Unique to ZIQBYZSX, ZSXBYZGP, ZIQBYZGP (2) −R sq 5 + R sq 7 −.37982 + .56680	.18698
Unique to THREEWAY (3) −R sq 4 + R sq 7 −.28416 + .56680	.28664
Common to (1) and (2) −R sq 3 + R sq 5 + R sq 6 − R sq 7 −.10609 + .37982 + .13730 − .56680	−.15577
Common to (1) and (3) −R sq 2 + R sq 4 + R sq 6 − R sq 7 −.01970 + .28416 + .13730 − .56680	−.34234
Common to (2) and (3) −R sq 1 + R sq 4 + R sq 5 − R sq 7 −.23331 + .28416 + .37982 − .56680	−.13613
Common to (1), (2), and (3) R sq 1 + R sq 2 + R sq 3 − R sq 4 − R sq 5 .23331 + .28416 + .10609 − .28416 − .37982 −R sq 6 + R sq 7 −.13730 + .56680	.38908

Table 5

Commonality Analysis Results

Partition	Set #1	Set #2	Set #3
Unique to ZSEX, ZGROUP, ZIQ	.42590		
Unique to ZIQBYZSX, ZIQBYZGP, ZSXBYZGP		.18698	
Unique to THREEWAY			.28664
Common to ZSEX, ZGROUP, ZIQ & ZIQBYZSX, ZIQBYZGP, ZSXBYZGP	-.15577	-.15577	
Common to ZSEX, ZGROUP, ZIQ & THREEWAY	-.34234		-.34234
Common to ZIQBYZSX, ZIQBYZGP, ZSXBYZGP & THREEWAY		-.13613	-.13613
Common to all three sets	.38908	.38908	.38908
Sum of partitions	.31687	.28416	.19725
r^2 of predictor with dependent variable	31.69%	28.42%	19.73%

Table 6
MANOVA Analysis

Source	Lambda	F	Hypoth. df	Error df
Three-way	.85	.63	2	7
GROUP by SEX	.97	.11	2	7
OVAIQ by SEX	.96	.16	2	7
OVAIQ by GROUP	.96	.15	2	7
SEX	.94	.22	2	7
GROUP	.69	1.54	2	7
OVAIQ	.67	1.75	2	7

Table 7

Composite Variables Using Canonical Function Weighting

Case	Composite I[1]	Composite II[2]
1	-.80	-1. 3
2	-.36	1.00
3	-.13	-.21
4	-.16	.66
5	1.04	.41
6	.79	-1.50
7	-1.87	.65
8	-1.85	-1.11
9	.46	.99
10	1.48	-.69
11	-.10	-1.08
12	-.15	1.55
13	.83	-.15
14	.15	.38
15	-.71	.82
16	1.38	.14

[1]Composite variables are computed using canonical function coefficient weights of.94(ZDV1) and .78(ZDV2). For example, for case 1, -.80 = [.94(-1.92)] + [.78(1.30)] = -1.8048 + 1.014.

[2]Composite variables are computed using canonical function coefficient weights of .49(ZDV1) and -.78(ZDV2).

Table 8

Prediction of Composite #1 Scores

Using Alternate Predictor Variable Combinations

Predictor set	Variable(s) in set	$R_c{}^2$
1	ZSEX	.02700
2	ZGROUP	.21491
3	ZIQ	.27817
4	ZGROUP, ZSEX	.24191
5	ZIQ, ZSEX	.32516
6	ZGROUP, ZIQ	.38281
7	ZGROUP, ZIQ, ZSEX	.42573

Table 9

Calculations of Unique Variance Partitions

Partition	Result
Unique to ZSEX -Rc sq 6 + Rc sq 7 -.38281 + .42573	.04292
Unique to ZGROUP -Rc sq 5 + Rc sq 7 -.32516 + .42573	.10057
Unique to ZIQ -Rc sq 4 + Rc sq 7 -.24191 + .42573	.18382
Common to ZGROUP and ZSEX -Rc sq 3 + Rc sq 5 + Rc sq 6 - Rc sq 7 -.27817 + .32516 + .38281 - .42573	.00407
Common to ZIQ and ZSEX -Rc sq 2 + Rc sq 4 + Rc sq 6 - Rc sq 7 -.21491 + .24191 + .38281 - .42573	-.01592
Common to ZGROUP and ZIQ -Rc sq 1 + Rc sq 4 + Rc sq 5 - Rc sq 7 -.02700 + .24191 + .32516 - .42573	.11434
Common to ZGROUP, ZSEX, and ZIQ Rc sq 1 + Rc sq 2 + Rc sq 3 - Rc sq 4 .02700 + .21491 + .27817 - .24191 -Rc sq 5 - Rc sq 6 + Rc sq 7 -.32516 - .38218 + .42573	-.00344

Table 10

Multivariate Commonality Analysis Results

Partition	Set #1	Set #2	Set #3
Unique to ZSEX	.04292		
Unique to ZGROUP		.10057	
Unique to ZIQ			.18382
Common to ZGROUP/ZSEX	.00407	.00407	
Common to ZIQ/ZSEX	-.01592		-.01592
Common to ZGROUP/ZIQ		.11434	.11434
Common to ZGROUP/ZIQ/ZSEX	-.00344	-.00344	-.00344
Sum of Partitions	.02763	.21554	.27880
r^2 of predictor with canonical composite scores	2.76%	21.55%	27.88%